

- A1. Find the area of the finite region bounded by the curves  $y = \frac{1}{x}$ ,  $y = \frac{1}{x^2}$ ,  $x = \frac{3}{4}$  and  $x = \frac{4}{3}$ . (Exact answer)
- A2. Find the radius of a sector (of a circle) with perimeter 12 and maximal area.
- A3. The plane vectors  $\vec{a}$  and  $\vec{b}$  have a common initial point on the line  $y = 4$ , and the endpoints  $(0, 2)$  and  $(0, -1)$ , respectively. Find the initial point in question, when  $a_b$  (the scalar projection of  $\vec{a}$  on  $\vec{b}$ ) is  $2/3$  of the length of  $\vec{b}$ .
- A4. The number of bacteria in a culture of bacteria is doubled every hour, if the bacteria are given a nutrient solution. If no nutrient solution is given, the number of bacteria decreases by  $p\%$  per hour. During an experiment nutrient solution was given from the time  $t = 0$  until the time  $t = 3$  h, after which time no nutrient was given. The number of bacteria is observed hourly. The first observation of a number smaller than at the initial time  $t = 0$  occurred at the time  $t = 11$  h (that is 11 hours after the initial time  $t = 0$ ). In which interval can the percentage number  $p$  lie on the basis of the observations? (Give the endpoints of the interval with one decimal.)
- A5. The time  $t$  ( $t$  days) between two strong earthquakes (strength over 3.5 on the Richter scale) is a random variable with exponential density function

$$f(t) = \begin{cases} \frac{1}{90} e^{-t/90} & , \text{for } t \geq 0 \\ 0 & , \text{for } t < 0 \end{cases} .$$

Find the probability that the following strong earthquake

- a) will take place within 7 days,
- b) will not take place during the first 5 weeks,

after the occurrence of a strong earthquake. (Give the probabilities with three decimals.)

- A6. A vessel contains a continually consumed mixture of ethanol and water. In the beginning, when the vessel is full, the mixing ratio of ethanol and water is 1:4. From here on, the vessel is always filled again, when half of its contents is consumed, by adding ethanol and water in the mixing ratio 1:9. Find the ethanol content of the mixture after the  $k$ th refilling. Which values does this content approach, when the number of refillings grows towards infinity?