HUT, TUT, LUT, OU, ÅA/Engineering departments
Mathematics examination 29.5.2001

Series A

Instructions: Reserve **a separate page for each problem**. Give your **solutions in a clear form including intermediate steps**. Write a clean copy of the solution if needed. **Cross out discarded solutions** (only the weaker one of two solutions on the same problem will be taken into account).

A1. A paperboy has two different alarm clocks. The better one works with probability 0.95 and the poorer one with probability 0.60 (independently of each other). The paperboy sets both clocks. What is the probability that

a) both clocks ring,
b) at least one of the clocks rings,
c) only one of the clocks rings?

A2. Find all solutions of the equation

$$\frac{4|x+1|}{x-2} = \frac{7}{|x-5|}.$$

A3. A carpenter makes pentagonal frames (Figure 1) from thin battens. The area of the frames is $A = 6.00\ m^2$. Each frame consists of a rectangle with breadth $L = 2.85\ m$ and height h, and an isosceles triangle with legs s. How should h and s be chosen, if we want to minimize the batten consumption? (Answer in metres to two decimals)

A4. At a T-junction a traffic count was made (Figure 1). During the count 370 cars arrived from direction A, while 460 cars left in direction A. Correspondingly, 410 cars arrived from direction B, while 530 cars left in direction B. It was further observed that $n_{CA} = 190$ of the cars arriving from direction C turned to direction A. (At the junction the cars either drive straight ahead or make a normal turn; U-turns are forbidden).

a) Let x be the number of cars that arrived to the junction from direction C. What is the least possible value of x?
b) What value for x do we obtain, if we further know that $n_{AB} = 120$ of the cars arriving from direction A continued straight ahead?

Motivate your solution carefully.

A5. Let $F(x)$ be the antiderivative (primitive) of the function

$$f(x) = \begin{cases} 7 + x + \cos(\pi x) & \text{if } x < 1 \\ 7 & \text{if } x \geq 1 \end{cases}$$

that satisfies $F(-1) = 0$. Find $F(0)$ and $F(\pi)$.

A6. The cancer tissue of a patient grows naturally $p\%$ per week. Every week he gets a radiation dose that destroys (immediately) a grammes of the cancer tissue. When the radiation treatment begins, the size of the cancer tumour is M grammes.

How large can M be at most, if one wants the tumour to be totally destroyed n weeks after the beginning of the treatment?

What is the value of M (in whole grammes) at most, if $p = 2$ (%), $a = 4$ (grammes) and $n = 28$ (weeks)?

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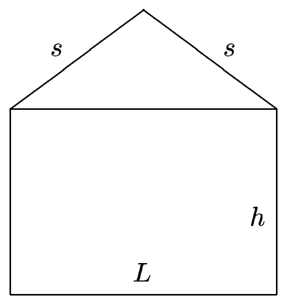

Figure 1

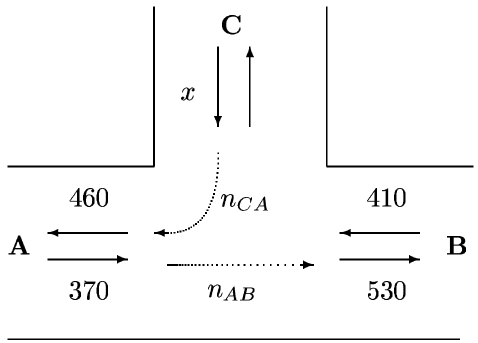

Figure 2