

Entrance examinations in mathematics May 30, 2006

Instructions. Place *each answer on a separate page*. Pay attention to the clarity of your answers, if needed, rewrite the answer. Include *the intermediate steps of the calculation* into your answer. Mark *any solution abandoned by crossing it over*, since only the worst of several alternative answers to one given problem will be credited.

A1. For which values of the parameter p does the equation $x^2 - 2px + p^2 - 4p + 16 = 0$ have the solution $x = 3$? What is then the other solution to the equation?

A2. Determine the maximum and the minimum value of the function

$$f(x) = \sin^3 x - 3 \sin x + \frac{28x}{11}$$

on the interval $[0, 7]$. (Give the answers with an accuracy of three decimals.)

A3. Three friends play a game in which one picks blind-folded from a bag white and black balls. In the bag there are four black balls and one white ball. The player whose turn it is picks one ball. If the ball is white the player has won; otherwise the ball is returned to the bag and the next player gets the turn. The turn rotates until the white ball is picked.

- a) What is the probability that the game ends before any of the players has picked twice?
- b) Let the players be $A, B,$ and $C,$ in this order. What is each player's probability of winning the game?

(Give the answers with an accuracy of three decimals.)

A4. For which values of the constant $k, k < 2,$ does it hold that

$$\int_k^2 |2x + 3| dx = \left| \int_k^2 (2x + 3) dx \right| \quad ?$$

A5. The indecisive spanworm Willy moves back and forth along a branch of length 1. Having turned towards the tip of the branch, it always moves 80 % of the distance between the turning point and the tip and then turns back towards the crotch of the branch. Moving towards the crotch Willy moves 60 % of the distance between the turning point and the crotch, and then again turns towards the tip.

Once, having just turned moving towards the tip, Willy's distance from the crotch is x_0 . When Willy the n^{th} time thereafter turns towards the tip, the distance is $x_n, n > 0$.

- a) Determine the distance x_n as a function of the distance x_{n-1} .
- b) Determine the distance x_n as a function of the distance x_0 .
- c) What is x_0 , when we know that Willy turns to a given direction always at the same spot? (Give the answer with an accuracy of two decimals.)

A6. Consider a unit circle centered at the origin. Let K be an arc on the circle in the first quadrant of the xy -plane. The line segments drawn from the origin to the end points of the arc K form angles α and β with the positive x -axis, $0 \leq \alpha < \beta \leq \frac{\pi}{2}$. Denote A the region between the arc K and x -axis as in Figure 1. Respectively, denote B the region between the arc K and y -axis. Show that the sum of the area of A and the area of B has the same numerical value as the length of the arc K .

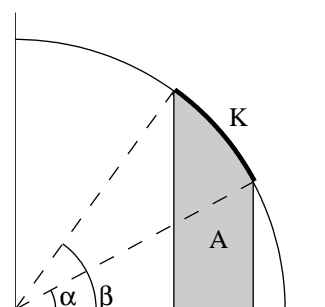


Figure 1